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Extended pairing model revisited

J.P. Draayer[1,a], Feng Pan[2], and V.G. Gueorguiev[1]

[1] Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA
[2] Department of Physics, Liaoning Normal University, Dalian, 116029, PRC

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Abstract. The mean-field plus extended pairing model proposed by the authors for describing well-deformed nuclei (F. Pan, V.G. Gueorguiev, J.P. Draayer, Phys. Rev. Lett. **92**, 112503 (2004)) is revisited. Eigenvalues of the model can be determined by solving a single transidental equation. Results to date show that even through the model includes many-body interactions, the one- and two-body terms continue to dominate the dynamics for small values of the pairing strength; however, as the strength of the pairing interaction grows, the higher-order terms grow in importance and ultimately dominate. Attempts to extend the theory to the prediction of excited zero plus states did not produce expected results and therefore requires additional consideration.

PACS. 21.10.Dr Binding energies – 71.10.Li Pairing interactions in model systems – 21.60.Cs Shell model

Pairing is an important residual interaction in nuclear physics. Much attention and progress, building on Richardson's early work [1] and various extensions to it based on the Bethe ansatz, has been made in the past few years. Solutions are provided by a set of non-linear Bethe Ansatz Equations (BAEs) [2]. Though these applications show that the pairing problem is exactly solvable, solutions of the BAEs are not trivial. This limits the applicability of the methodology to relatively small systems; it cannot be applied to large systems such as well-deformed nuclei.

As an extension of the standard pairing interaction, we constructed the following new Hamiltonian:

$$\hat{H} = \sum_{j=1}^{p} \epsilon_j n_j - G \sum_{i,j=1}^{p} a_i^+ a_j - G \sum_{\mu=2}^{p} \frac{1}{(\mu!)^2} \times \sum_{i_1 \neq \cdots \neq i_{2\mu}} a_{i_1}^+ \cdots a_{i_\mu}^+ a_{i_{\mu+1}} \cdots a_{i_{2\mu}}, \quad (1)$$

where p is the total number of levels considered, $G > 0$ is the pairing strength, ϵ_j single-particle energies taken, for example, from the Nilsson model, $n_j = c_{j\uparrow}^\dagger c_{j\uparrow} + c_{j\downarrow}^\dagger c_{j\downarrow}$ is the fermion number operator for the j-th level, and $a_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$ ($a_i = (a_i^+)^\dagger = c_{i\downarrow} c_{i\uparrow}$) are pair creation (annihilation) operators. The up and down arrows refer to time-reversed states. Since each level can only be occupied by one pair due to the Pauli Principle, the operators a_i^+, a_i, and n_i satisfy the hard-core boson algebra: $[a_i, a_j^+] = \delta_{ij}(1 - n_i)$, $[a_i^+, a_j^+] = 0 = (a_i^+)^2$.

Besides a mean-field and standard pairing, the interaction includes multi-pair hopping terms that allow pairs to simultaneously scatter (hop) between and among different levels. With this extension in place, the model can be shown to be exactly solvable [3].

If $|j_1, \cdots, j_m\rangle$ is the pairing vacuum, where $j_1, \cdots, j_m$ are levels occupied by single nucleons, thus blocked by the Pauli principle, then the k-pair eigenstate is

$$|k; \zeta; j_1 \cdots j_m\rangle = \sum_{i_1 < \cdots < i_k} C_{i_1 \cdots i_k}^{(\zeta)} a_{i_1}^+ \cdots a_{i_k}^+ |j_1 \cdots j_m\rangle, \quad (2)$$

where $C_{i_1 i_2 \cdots i_k}^{(\zeta)}$ are expansion coefficients that are to be determined. It is assumed that the indices $j_1, \cdots, j_m$ should be excluded from the summation.

Since the formalism for even-odd systems is similar, we focus on the even-even seniority zero case where the excitation energies $E_k^{(\zeta)}$ and expansion coefficients $C_{i_1 i_2 \cdots i_k}^{(\zeta)}$ of the k-pair eigenstates are given by

$$E_k^{(\zeta)} = \frac{2}{x^{(\zeta)}} - G(k-1), \quad (3)$$

$$C_{i_1 i_2 \cdots i_k}^{(\zeta)} = \frac{1}{1 - x^{(\zeta)} \sum_{\mu=1}^{k} \epsilon_{i_\mu}}, \quad (4)$$

and the variable $x^{(\zeta)}$ is determined by

$$\frac{2}{x^{(\zeta)}} + \sum_{1 \le i_1 < i_2 < \cdots < i_k \le p} \frac{G}{(1 - x^{(\zeta)} \sum_{\mu=1}^{k} \epsilon_{i_\mu})} = 0. \quad (5)$$

[a] Conference presenter; e-mail: Draayer@phys.lsu.edu

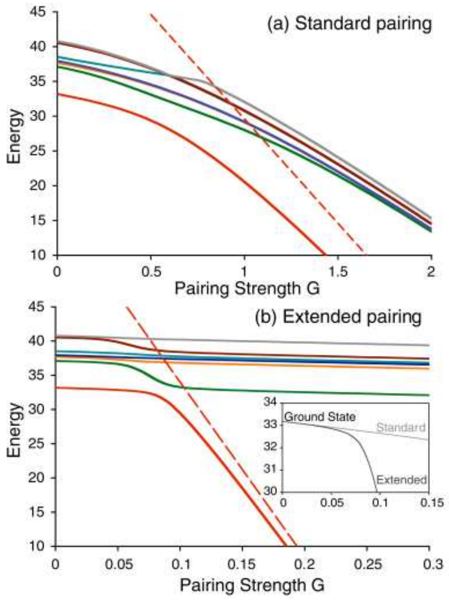

Fig. 1. (a) Spectral structure of the standard pairing interaction, and (b) spectral structure of the extended pairing interaction given by eq. (1), as functions of the pairing interaction strength G for $k = 5$ pairs for a system with $p = 10$ levels, where the single-particle energies and G are given in arbitrary units. The straight dash line is the expectation value of the Hamiltonian in the pure pairing ($\epsilon_i = 0$) ground state.

The label $\zeta = 1, 2, 3, \cdots$ in this expression can be understood as the ζ-th solution of (5). For even-odd systems the level j_s occupied by the single nucleon should also be excluded from the summation in (2) and the single-particle energy term ϵ_{j_s} contributing to the eigenenergy from the first term of (1) should be included. Although these eigenstates (2) are not normalized, they can be normalized easily; the eigenstates (2) with different roots given by (5) are of course mutually orthogonal. Extensions of this to many broken-pair cases are straightforward.

To gain a better understanding of the extended pairing theory, we considered an example of $p = 10$ levels with single-particle energies given by $\epsilon_i = i + \chi_i$ for $i = 1, 2, \cdots, 10$, where χ_i are random numbers within the interval $(0, 1)$ and the pairing strength G varies from 0.01 to 0.10. Figure 1 shows the lowest few energies of the standard and extended pairing models for this case. It is clear that there are essential differences in the spectra. As shown in fig. 1(b), the extended pairing model rapidly develops a paired ground-state configuration and the transition from mean-field eigenstates to pairing eigenstates is sharp and rapid, while standard pairing, fig. 1(a), exhibits a slower and smoother transition. The differences in the spectra is a distinguishing characteristic that can be used to explore cases where the extended pairing concept might be more relevant and appropriate than the standard pairing model.

Since there are higher order terms involved in (1), it is important to know whether the dynamics is still dominated by the one- and two-body interactions or if the presence of the higher-order terms alters this picture. To explore this, we calculated as a function of G the expectation value of each higher order term $\langle V_\mu \rangle$ defined by:

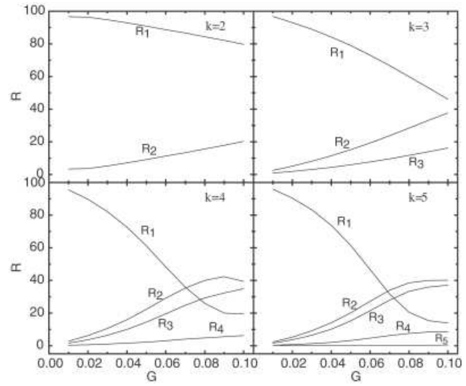

Fig. 2. Ratios $R_\mu(\%)$ with $\mu = 1, 2, \cdots, 5$ as a function of the pairing interaction strength G for $k = 2, \cdots, 5$ for $p = 10$ levels.

$$V_1 = \sum_{i,j} a_i^+ a_j ,$$

$$V_\mu = \frac{1}{(\mu!)^2} \sum_{i_1 \neq \cdots \neq i_{2\mu}} a_{i_1}^+ \cdots a_{i_\mu}^+ a_{i_{\mu+1}} \cdots a_{i_{2\mu}}$$

with $\mu = 2, 3, \cdots$, for k-pair ground states. We calculated the ratio $R_\mu = \langle V_\mu \rangle / \langle V_{\rm total} \rangle$, where $\langle V_{\rm total} \rangle$ is the sum of all terms above. The results, which are shown in fig. 2, indicate that the two-body pairing interaction (V_1) dominates the dynamics of the system for small interaction strength G. With increasing interaction strength, the system is driven increasingly by the higher-order terms.

Returning to eq. (3), it is natural to consider excited as well as ground state solutions. Once the coupling strength is fixed from the ground state, excited states can also be calculated. However, initial calculations suggest they do not agree well with the experimentally observed values; that is, the dependence of the strength on the particle number that is required to make the extended theory reproduce first excited states seems to be different than for the ground state. This poses a dilemma; namely, whether or not the agreement for ground states was fortuitous rather than fundamental. This and other matters, such as whether or not the extended Hamiltonian has a special coherent-state–like solution, remain under investigation.

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References

1. R.W. Richardson, Phys. Lett. **3**, 277 (1963); R.W. Richardson, N. Sherman, Nucl. Phys. **52**, 221 (1964).

2. J. Dukelsky, V.G. Gueorguiev, P. Van Isacker, nucl-th/0406001.
3. F. Pan, V.G. Gueorguiev, J.P. Draayer, Phys. Rev. Lett. **92**, 112503 (2004).
4. P. Möller, J.R. Nix, K.L. Kratz, At. Data Nucl. Data Tables **66**, 131 (1997).
5. G. Audi, O. Bersillon, J. Blachot, A.H. Wapstra, Nucl. Phys. A **624**, 1 (1997).